Learning to solve trigonometry problems: A comparative study of the analogy, worked-example and problem-solving approaches

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Trigonometry problems: a challenge

- Students struggled to solve $\sin 30^{o} = \frac{x}{5}$ despite having learned how to solve a similar problem such as $\frac{x}{2} = 3$.
- Students struggled to solve $\cos 20^{0} = \frac{6}{x}$ despite having learned how to solve a similar problem such as $\frac{8}{x} = 2$

Current approach

- Mathematics teachers tend to drill students to solve trigonometry problems such as $\sin 30^{\circ} = \frac{x}{5}$ by instructing them to multiply both sides by 5.
- For a conceptually more difficult trigonometry problem such as $\cos 20^{\circ} = \frac{6}{x}$ in which the pronumeral is a denominator, students are taught to *swap* the x with $\cos 20^{\circ}$ to solve the problem.

Issue with the current approach

- The current approach falls short of directing students' attention to the underlying concepts involved in learning how to solve trigonometry problems.

- It does not attempt to make a link to prior knowledge of solving equations with a fraction on which to build the skill on solving trigonometry problems.

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Learning by analogy

- Learning by analogy, underpinned by structure mapping theory, predicts that successful mapping of the structural elements of a new problem (target) with a learned problem (source) is likely to result in analogical transfer (Gentner, 1983; Richland & McDonough, 2010).

Learning by analogy (Cont.)

Equation with a fraction (source problem)	Trigonometry problem (target problem)
$\frac{x}{2} = 3$	$\sin 30^0 = \frac{x}{5}$
$\frac{8}{x} = 2$	$\cos 20^0 = \frac{6}{x}$

Which type of cognitive load will be involved in the analogy approach?

- Germane cognitive load is expected to increase when mapping structurally similar elements between an equation with a fraction (source problem) and a trigonometry problem (target problem), which is likely to benefit learning of the trigonometry problems.

The present study

- We compared the analogy, worked example and problem-solving approaches in facilitating learning of trigonometry problems from a cognitive load theory perspective.

Analogy approach

Example 1

Study and compare Part (I) and Part (II). Use your calculator to verify the answers.

Part (I)		Part (II)	
$7 = \frac{77}{d}$	[$\div d$ becomes $\times d$]	$\cos 7° = \frac{36}{x}$	[$\div x$ becomes $\times x$]
$d \times 7 = 77$	[$\times$ 7 becomes $\div 7$]	$x \times \cos 7° = 36$	[$\times \cos 7°$ becomes $\div \cos 7°$]
$d = \frac{77}{7} = 11$		$x = \frac{36}{\cos 7°} = 36.27$	

Equation 1

$\cos 43° = \frac{60}{x}$

Worked example approach

Example 1

$$\cos 7° = \frac{36}{x} \qquad [\div x \text{ becomes } \times x]$$

$$x \times \cos 7° = 36 \qquad [\times \cos 7° \text{ becomes } \div \cos 7°]$$

$$x = \frac{36}{\cos 7°} = 36.27$$

Equation 1

$$\cos 43° = \frac{60}{x}$$

Problem-solving approach

Equation 1

$$\cos 7° = \frac{36}{x}$$

Equation 2

$$\cos 43° = \frac{60}{x}$$

Hypotheses

- Hypothesis 1: Performance on post-test and the concept test would follow the order: analogy group > worked example group > problem-solving group.

- Hypothesis 2: Mental effort rating would follow the order: problem-solving approach > analogy approach > worked example approach.

- Hypothesis 3: The correlation between post-test and concept-test would be positive for the analogy and worked example groups but not the problem-solving group.

Experimental procedure

- Sample: Sixty three students (mean age =15) who had basic knowledge of the trigonometric ratio.
- Pre-test (10 minutes)
- Acquisition phase (20 minutes)
    - Studied an instruction sheet (5 minutes)
    - Completed acquisition problems (15 minutes).
    - Rated the mental effort invested on a Likert scale
- Post-test (10 minutes)
- Concept test (5 minutes)

Materials

- Pre-test had identical content as the post-test (16 problems), both of which have similar problem structure as the acquisition problems.
- Acquisition problems: 12 example-problem pairs (analogy group, worked example group), 24 problems (problem solving group)

Test materials (cont.)

- Concept test (8 pairs):

Equation	Equation	Circle 'Yes 'or 'No'	Reason
(i) $sin11° = \frac{35}{x}$	(ii) $35 = sin11° \times x$	Yes No	
(a) $sin11° = \frac{35}{x}$	(b) $x = \frac{35}{sin11°}$	Yes No	

Results

Table 1

Performance Outcomes of Practice Problems, Mental Effort, Pre-test, Post-test and Concept Test

	Problem solving approach $n = 20$ *M (SD)*	Worked example approach $n = 19$ *M (SD)*	Analogy approach $n = 21$ *M (SD)*
Practice problems	0.84 (0.33)	0.94 (0.07)	0.94 (0.09)
Mental Effort	5.17 (2.09)	4.47 (1.61)	3.76 (1.51)
Pre-test	0.34 (0.35)	0.52 (0.30)	0.39 (0.33)
Post-test	0.83 (0.26)	0.85 (0.19)	0.74 (0.27)
Concept test	0.58 (0.27)	0.70 (0.33)	0.64 (0.30)

Note. We calculated proportion correct solutions for the practice problems, pre-test, post-test and concept test.

Results (Cont.)

One way ANOVA on pre-test, practice problems, post-test concept test and mental effort

All were nonsignificant except the mental effort, $F(2, 55) = 3.16$, $p = 0.05$. A Tukey post-hoc test revealed a significant difference between the problem-solving group ($M = 5.17$) and the analogy group ($M = 3.76$), $p = 0.04$, but not between other groups.

Thus, hypothesis 1 is not supported and hypothesis 2 is partially supported.

Results (cont.)

Correlation: post-test and concept test

- Analogy group (r = .597, n = 21, p = .004)
- Problem-solving group (r = -.179, n = 20, p = .449)
- Worked example group, (r = .216, n = 19, p = .374)

Thus, hypothesis 3 is partially supported.

Consideration from the cognitive theory perspective

Problem-solving approach

- Imposed the highest mental effort

Worked-example approach

- Mental effort imposed was mid-way between the problem-solving and analogy approaches

Analogy group

- Imposed the lowest mental effort

Implication for mathematics education

Nonsignificant correlation between post-test and concept test: Problem-solving and worked example approaches

- Students could solve the trigonometry problems but they may not understand the underlying concepts.

Significant correlation between post-test and concept test: Analogy group

- Learning via analogical reasoning is liken to 'deliberate practice' (van Gog, Ericsson, Rikers, & Paas, 2005) which help students to understand the underlying concepts.

Future research

- Make the process of mapping more explicit in the analogy approach (e.g., ask the learner to conduct one-to-one mapping of similar elements between the source and target problems).

- Using students of varying ability level.

- Experimental design: 3 x (method: analogy, worked example, problem solving) x 2 (level of element interactivity: $\sin 30^0 = \frac{x}{5}$, $\cos 20^0 = \frac{6}{x}$).