# Achieving optimal best: The Use of Cognitive Load Theory in Mathematical Problem Solving

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# Introduction

Successful schooling entails more than just academic achievement. There are other *facets* and *attributes* for consideration, for example:

1. Heightened motivational states.
2. Mastery and engagement of deep learning.
3. Enriched well-being experiences.

It is important for educators to focus on other *school-based* and *achievement-related outcomes*. We need to consider theoretical orientations, pathways, strategies, and programs that could facilitate and foster the aforementioned facets and attributes.

In this presentation, we introduce the *Framework of Achievement Bests* (Phan, Ngu, & Yeung, In press), which focuses on the importance of *Realistic achievement best* and *Optimal achievement best* in mathematical problem solving .

# The Framework of Achievement Bests

The ***Framework of Achievement Bests*** emphasizes two major theoretical tenets (Figure 1):

1. The *Realistic-Optimal Achievement Best Continuum*, which focuses on different types of best practice.
2. The *Process of Optimization*, which focuses on an internal psychological process of optimal functioning of exceptional best practice.

The Framework of Achievement Bests, in this case, focuses on the positive nature of best practice. What is the best that a student can do in, say, algebra? Likewise, what is the best that an individual can do in terms of his/her employment prospects. Achievement bests, in this sense, emphasize on the importance of aspiration for success, personal endeavor, and maximization of capability.

# The Framework of Achievement Bests

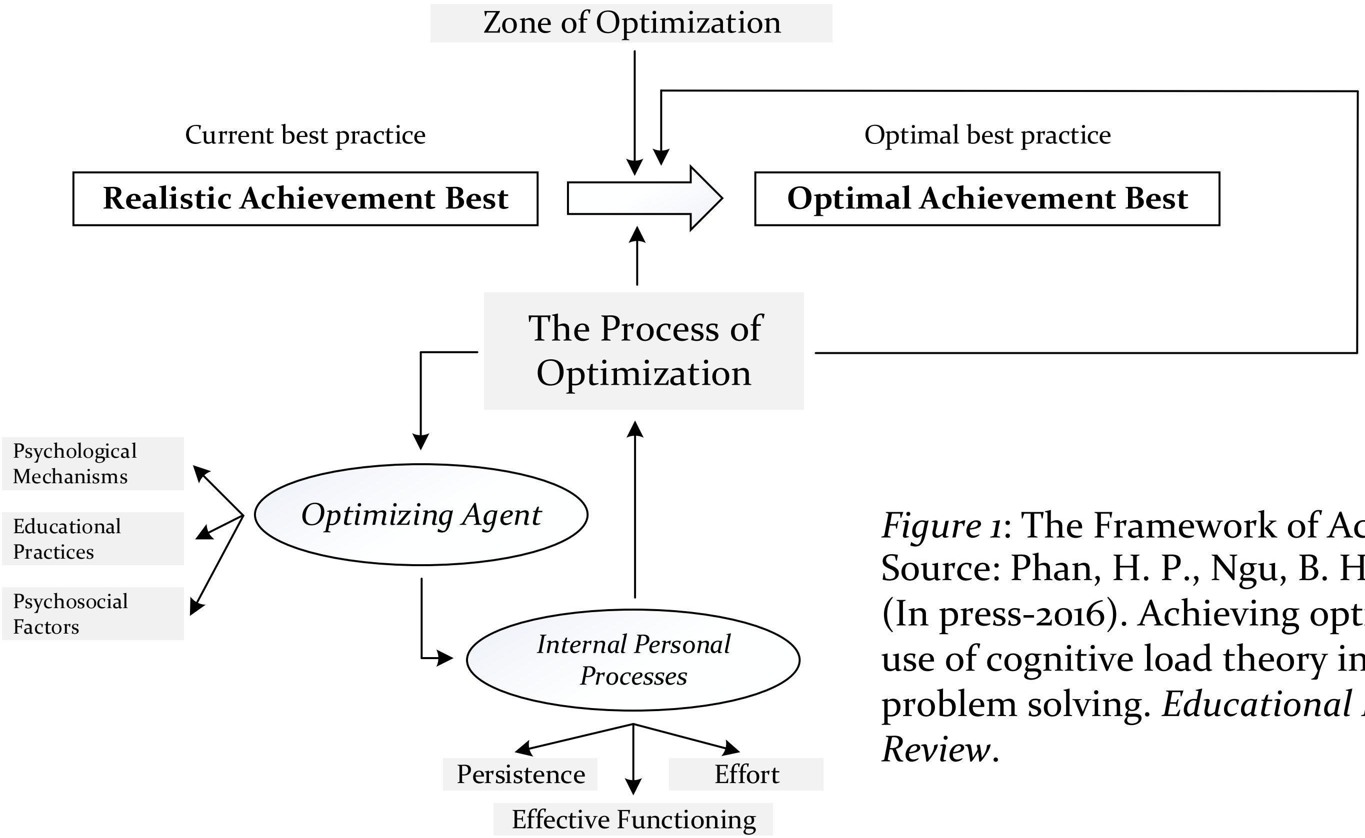


*Figure 1*: The Framework of Achievement Bests
Source: Phan, H. P., Ngu, B. H., & Yeung, A. S. (In press-2016). Achieving optimal best: The use of cognitive load theory in mathematical problem solving. *Educational Psychology Review*.

# The Realistic-Optimal Achievement Best Continuum

The ***Realistic-Optimal Achievement Best Continuum*** indicates two major types of achievement best, namely:

1. *Realistic Achievement Best*, which is defined as "an individual's actual competence at any given time to learn and/or to solve a problem" (Phan, Ngu, & Williams, 2016, p. 163).

2. *Optimal Achievement Best*, which is defined as "an individual's striving to demonstrate and/or to seek mastery in competence at any given time, reflecting his/her fullest capacity" (Phan, Ngu, & Williams, 2016, p. 163).

# Relationship between Realistic and Optimal Achievement Bests

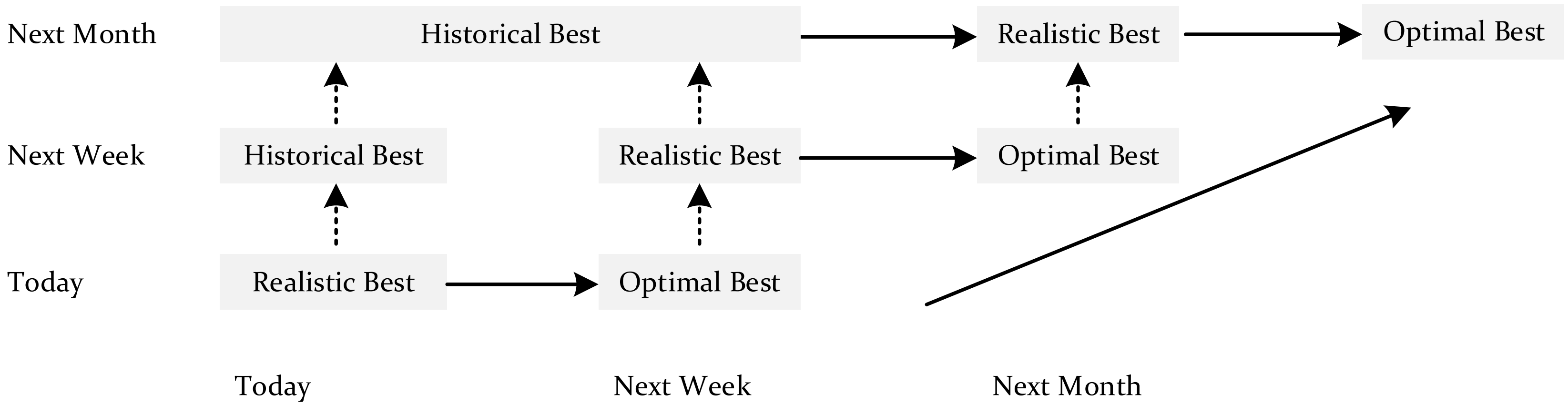


*Figure 2*: Relationships between Realistic and Optimal Achievement Bests

Source: Phan, H. P., & Ngu, B. H. (In Review). Understanding the Framework of Achievement Bests. *The Journal of Educational Research*.

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# An Example of Realistic and Optimal Achievement Bests in Mathematics

How does the realistic-optimal best continuum fit in with the context of academic learning? We consider an example of mathematics, as shown:

Zone of Optimization

**Realistic Best**

*Demonstration of competence*
Due to lack of understanding of the relation between *find-part*, *find-percent* and *find-whole* percentage problems, the student's ability to solve these problems would follow this sequential order: find-part > find-percent > find-whole.

1. *Find-part*
   The cost of a new graphite tennis racquet is $1350. What is 8% of the cost of the new graphite tennis racquet?
2. *Find-percent*
   The cost of a new graphite tennis racquet is $1350. What percent of the new graphite tennis racquet is $108?
3. *Find-whole*
   I spent 8% of my money on a new graphite tennis racquet, which costs $108. How much money did I have before the purchase?

**Optimal Best**

*Demonstration of competence*
Excellent grasp of the relation between *find-part*, *find-percent* and *find-whole* percentage problems. The student would competently solve the three types of percentage problems.

1. *Find-part*
   The cost of a new graphite tennis racquet is $1350. What is 8% of the cost of the new graphite tennis racquet?
2. *Find-percent*
   The cost of a new graphite tennis racquet is $1350. What percent of the new graphite tennis racquet is $108?
3. *Find-whole*
   I spent 8% of my money on a new graphite tennis racquet, which costs $108. How much money did I have before the purchase?"

*The seeking of mastery of competence*

1. Ability to solve the transfer percentage problem, comprising three types of percentage problems. When Year 8 students at St. John School were surveyed about how they travelled to school each day, it was found that 25% walk, 59% come by bus and the remaining 25 students are driven to school.
   a. If 39 students walk to school, how many Year 8 students are there at St John School? [*find-whole*]
   b. Calculate the number of students who come by bus. [*find-part*]
   c. What percentage of students are driven to school? [*find-percent*]
2. Is there more than one strategy to solve the three types of percentage problems?

*Figure 3*: An Example of Realistic-Optimal Achievement Bests Continuum.

Source: Phan, H. P., Ngu, B. H., & Yeung, A. S. (In press-2016). Achieving optimal best: The use of cognitive load theory in mathematical problem solving. *Educational Psychology Review*.

# Element Interactivity and Instructional Approaches

Differential element interactivity exists between the unitary, unitary-pictorial, equation and equation-pictorial approaches for learning how to solve the percentage problem such as:

*I spent 8% of my money on a new graphite tennis racquet, which costs \$108. How much money did I have before the purchase*?"  (see Figure 2).

# The Unitary Approach

For example, the steps involved in the unitary approach are shown in Figure 4 (McSeveny et al., 2004, p. 76) :

| | | |
|---|---|---|
| *Step 1* | | 8% of my money = \$108 |
| *Step 2* | ∴ 1% of my money | \$108 ÷ 8 = \$13.50 |
| *Step 3* | ∴ 100% of my money | \$13.50 × 100 = \$1350 |

*Figure 4*: Unitary Approach

Source: Phan, H. P., Ngu, B. H., & Yeung, A. S. (In press-2016). Achieving optimal best: The use of cognitive load theory in mathematical problem solving. *Educational Psychology Review*.

In relation to element interactivity, in this case:

- Step 1 involves three elements (i.e., 8%, my money, \$108) and one concept (i.e., the relation between 8%, my money, and \$108 where the left side equals to the right side). These elements and concept need to be processed, simultaneously.
- Step 2 involves three elements (i.e., \$108, 8, \$13.50) and two concepts (i.e., \$108 represents 8%, and the sub-goal is 1% of my money). The learner will need to manipulate these elements and concepts, simultaneously.
- Step 3 involves three elements (i.e., \$13.50, 100, \$1350) and two concepts: (i.e., \$13.50 is equivalent to 1% of my money, and \$13.50 × 100 is equivalent to 100% of my money). Manipulating multiple elements (i.e., quantity, %) within and between the steps will result in high cognitive load.

# The Unitary Approach (Cont.)

Apart from this, the calculation of 1%, which is central to the unitary approach, will be problematic without a point of reference, such as 8% is equivalent to $108. Searching and integrating discrete sources (8% in step 1 and $108 in step 2) will result in extraneous cognitive load (Yeung, Jin, & Sweller, 1998).

Overall then, the combined effects of intrinsic and extraneous cognitive loads that are imposed in the solving of the mentioned problem are likely to render this unitary approach as ineffective for implementation and/or use.

# The Unitary-Pictorial Approach

The unitary-pictorial approach will share, similarly, the solution steps mentioned in the unitary approach. Figure 5 shows an example of the unitary-pictorial approach used to solve the same percentage problem. Processing the information in the diagram is expected to increase germane cognitive load. The diagram will align \$108 with 8%, and "my money" with 100%. The alignment not only will eliminate the *split-attention* effect (Yeung et al., 1998), but also will help the learner to form a mental representation of "my money" (100%), which is greater than \$108 (8%) based on proportional reasoning. The germane cognitive load is increased to elicit the concept of *proportional reasoning*, and allows learners to process fewer elements in the working memory (Carlson, Chandler, & Sweller, 2003). Moreover, the alignment between \$108 and 8% will serve as a point of reference for the calculation of "1% of my money". The unitary-pictorial approach, we expect, will incur lower element interactivity than the unitary approach, because the diagram not only eliminates the split-attention effect, but also elicits proportional schema, both of which will reduce the degree of element interactivity.

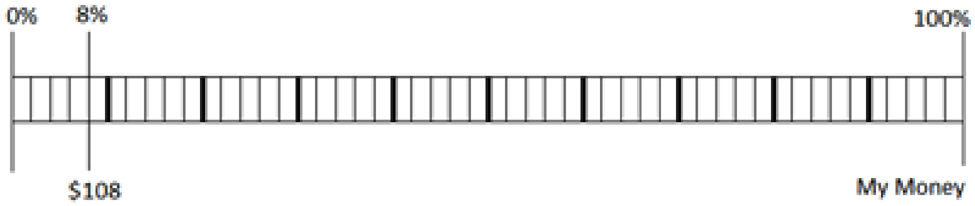


*Figure 5*: Unitary-Pictorial Approach

Source: Phan, H. P., Ngu, B. H., & Yeung, A. S. (In press-2016). Achieving optimal best: The use of cognitive load theory in mathematical problem solving. *Educational Psychology Review*.

# The Equation Approach

This instructional approach requires the learners to set up an equation to represent the problem situation, as depicted in Figure 6. There are a number of steps involved here, for example:

- Step 1 involves the creation of an unknown variable, $x$. The concept of a variable may pose a challenge if the learner has no prior knowledge of a variable in the context of problem solving.
- Step 2 involves three elements (i.e., 8%, $x$, \$108) and one concept (i.e., the relation between the quantity, percentage and the unknown, where the left side equals to the right side). The learner will need to process the three elements and concept, simultaneously.
- Step 3 involves three elements (i.e., $x$, \$108, 8%) and one concept (i.e., move ×8% from the left side of step 2 to become ÷ 8% on the right side of step 3). Interaction between the elements will occur on the right side, where ÷ 8% will interact with \$108, giving rise to the solution: \$1350.

# The Equation Approach (Cont.)

The equation approach will integrate relevant information (i.e., “my money”, 8%, $108) to form a symbolic representation of the problem situation in a single equation (Hegarty, Mayer, & Monk, 1995). Compared to the unitary and/or unitary-pictorial approach, the degree of element interactivity within and between the solution steps is relatively lower.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Step 1 | Create an unknown | Let $x$ be my money | | | | | |
| Step 2 | 8% of my money is $108 | | 8%× | $x$ | = | $108 | × 8% becomes ÷ 8% |
| Step 3 | Solve the equation | | | $x$ | = | $108 ÷ 8% | |
| | | | | $x$ | = | $1350 | |

*Figure 6*: Equation Approach

Source: Phan, H. P., Ngu, B. H., & Yeung, A. S. (In press-2016). Achieving optimal best: The use of cognitive load theory in mathematical problem solving. *Educational Psychology Review*.

# The Equation-Pictorial Approach

The diagram in this approach is identical to that of the diagram found in the unitary-pictorial approach, with the exception that $x$ replaces “my money”. The approach will integrate information portrayed in the diagram to form an equation that is based on proportional reasoning (Figure 7).

- Step 1 here is similar to Step 1 in the equation approach (i.e., create an unknown variable, $x$).
- Step 2 involves four elements (i.e., $x$, 100, 108, 8) and one concept (i.e., $x/100$ is proportional to 108/8, and the left side equals to the right side).
- Step 3 involves four elements (i.e., $x$, 108, 100, 8) and one concept (i.e., move ÷100 from the left side of step 2 to become ×100 on the right side of step 3). The interactions between the elements will occur on the right side, where ×100 will interact with 108/8, giving rise to the solution: $1350.

# The Equation-Pictorial Approach (Cont.)

The equation-pictorial approach differs from the equation approach in terms of the format of the equation ($x/100 = 108/8$ vs. $8\% \times x = 108$), but they share similar solution steps and thus the same degree of element interactivity. Investing germane cognitive load to process the diagram that depicts problem structure (i.e., alignment between \$108 with 8%, and $x$ with 100%) is likely to foster deep learning even though it may impose lower degree of element interactivity owing to the familiarity with the proportional schema.

| | | | |
|---|---|---|---|
| *Step 1* | Create an unknown | Let $x$ be my money | |
| *Step 2* | From the diagram, form an equation | $\frac{x}{100} = \frac{108}{8}$ | ÷100 becomes ×100 |
| *Step 3* | Solve the equation | $x = \frac{108}{8} \times 100$ | |
| | | $x = \$1350$ | |

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*Figure 7*: Equation-Pictorial Approach

Source: Phan, H. P., Ngu, B. H., & Yeung, A. S. (In press-2016). Achieving optimal best: The use of cognitive load theory in mathematical problem solving. *Educational Psychology Review*.

# Achieving Optimal Best for Percentage Problems

In essence, the equation-pictorial and equation approaches allow the learners to view the relation between find-part, find-percent, and find-whole problems based on the format of the equations (Figure 2). The critical feature of the format of equations is the relative position of the unknown variable, *x*, which defines the solution to the find-part, find-percent, and find-whole problems.

For the equation-pictorial approach, the format of equation, which is derived from the diagram, is as follows: (1) find-part, $\frac{x}{8}=\frac{1350}{100}$ (2) find-percent, $\frac{x}{108}=\frac{100}{1350}$ and (3) find-whole, $\frac{x}{100}=\frac{108}{8}$.

Regarding the equation approach, the format of the equation is as follows: (1) find-part, 8% × \$1350 = *x*, (2) find-percent, *x* ×\$1350 = \$108, and (3) find-whole, 8% × *x* = \$108.

From a cognitive load perspective, the relative position of the unknown variable, *x*, in the format of the equations does not alter the degree of element interactivity across the three types of percentage problems. Thus, learners are likely to view the three types of percentage problems to have equivalent complexity. Accordingly, they are able to extend their ability not only to solve the three types of percentage problems separately, but also to solve transfer problems that consist of find-part, find-percent and find-whole features in a single problem that is situated in real-life contexts.

# Achieving Optimal Best for Percentage Problems (Cont.)

For the unitary or unitary-pictorial approach, the point of reference to calculate the unit quantity is as follows: (1) find-part, 100% corresponds to $1350 (calculate 1%), (2) find-percent, $108 corresponds to $1350 (calculate $1), and (3) find-whole, 8% corresponds to $108 (calculate 1%).

As noted in Figure 4, the solution steps of the unitary approach do not indicate a point of reference. Thus, the learners may view the three types of percentage problems to have differential complexity. Consequently, the unitary or unitary-pictorial approach could have limited students' motivation to advance from realistic best to optimal best in the acquisition of skills to solve find-part, find-percent and find-whole percentage problems.

# The Process of Optimization

The ***process of optimization***, as part of the Framework of Achievement Bests, is significant for its explanatory account of how a person reaches optimal best practice from realistic best practice. This proposed process expands on from previous theoretical orientations (e.g., Vygotsky, 1978; Wigfield & Eccles, 2000), and makes an attempt to provide a detailed account of how the *zone of optimization* is fulfilled. The zone of optimization is defined, in this case, as the range or difference between level of best practice (e.g., realistic ↔ optimal).

Achievement of exceptional best practice, in this case, does not exist in isolation. Rather, as explained in Figure 1, there is an internal process that facilitates this achievement (e.g., realistic → optimal), namely: the 'triggering' of *internal personal processes* by *optimizing agents*.

1. Optimizing agents include: *psychological mechanisms* (e.g., personal self-efficacy), *educational practices* (e.g., appropriate pedagogical approach), and *psychosocial factors* (e.g., the impact of the home environment).

2. Internal personal processes include: *persistence*, *effort expenditure*, and *effective functioning*.

# An example of Optimization

How does the process of optimization occur? Consider, in this case, an example of mathematics learning.

1. An appropriate pedagogical approach that is used (e.g., the equation-pictorial approach ) triggers internal personal processes.
2. One personal process, in this case, involves a report of effective functioning, which emphasizes on the importance of efficiency and execution of course of action in a functional, regulated manner.
3. The sequential triggering of an optimizing agent (i.e., appropriate pedagogical approach) on internal personal processes (i.e., effective functioning), in turn, facilitates the achievement of optimal best from realistic best.

# Cognitive Load and Achievement Bests: In Totality

We have provided a conceptualization that depicts, in totality, the associations between instructional designs, cognitive load imposition, motivation, achievement bests, and mathematics competence (Figure 7). This conceptual model, noteworthy of research advancement, emphasizes on interrelations that are both positive (e.g., appropriate and effective instructional design → optimal achievement best; optimal achievement best ↔ mathematics competence) and negative (e.g., cognitive load imposition → mathematics competence), in nature.

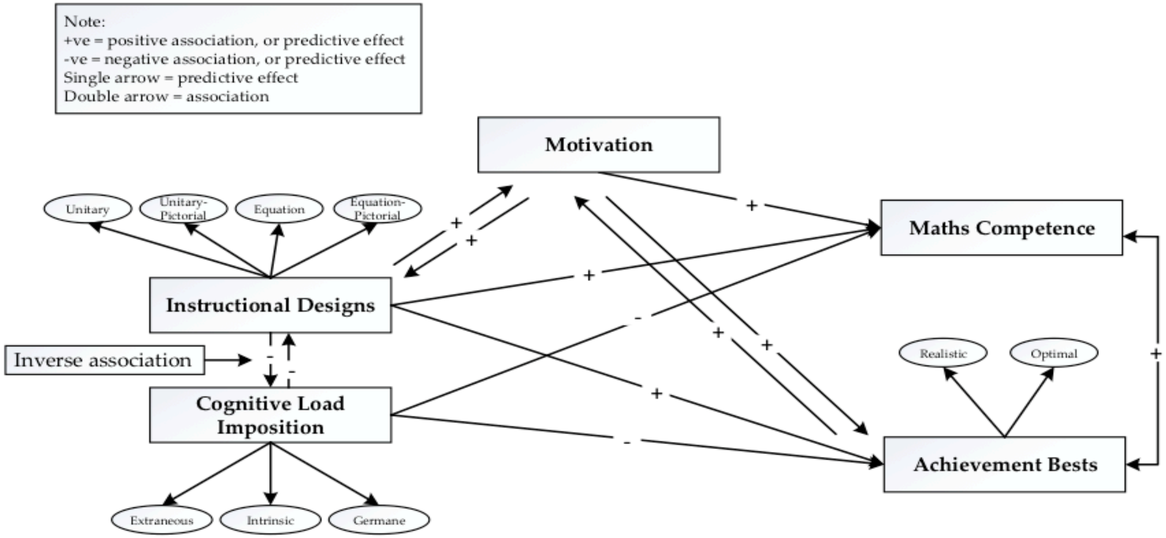


Figure 8: Instructional Designs, Cognitive Load Imposition, Motivation, Achievement Bests, and Mathematics Competence. Source: Source: Phan, H. P., Ngu, B. H., & Yeung, A. S. (In press-2016). Achieving optimal best: The use of cognitive load theory in mathematical problem solving. *Educational Psychology Review*.

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# Measurements of Achievement Bests

How do we measure and assess a person's realistic and/or optimal achievement best? The Optimal Outcomes Questionnaires (Phan, Ngu, & Williams, 2015), containing two subscales, rated on a 7-point rating scale, 1 (Always False) to 7 (Always True):

1. *Realistic Achievement Best* (eight items):

   "I am content with what I have accomplished so far for this subject (e.g., mathematics)" (Note: total scores ranged from 8 to 56)

2. *Optimal Achievement Best* (eight items):

   "I can achieve much more in this subject (e.g., mathematics) than I have indicated through my work so far"

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# Four Quadrants of Realistic and Optimal Achievement Bests

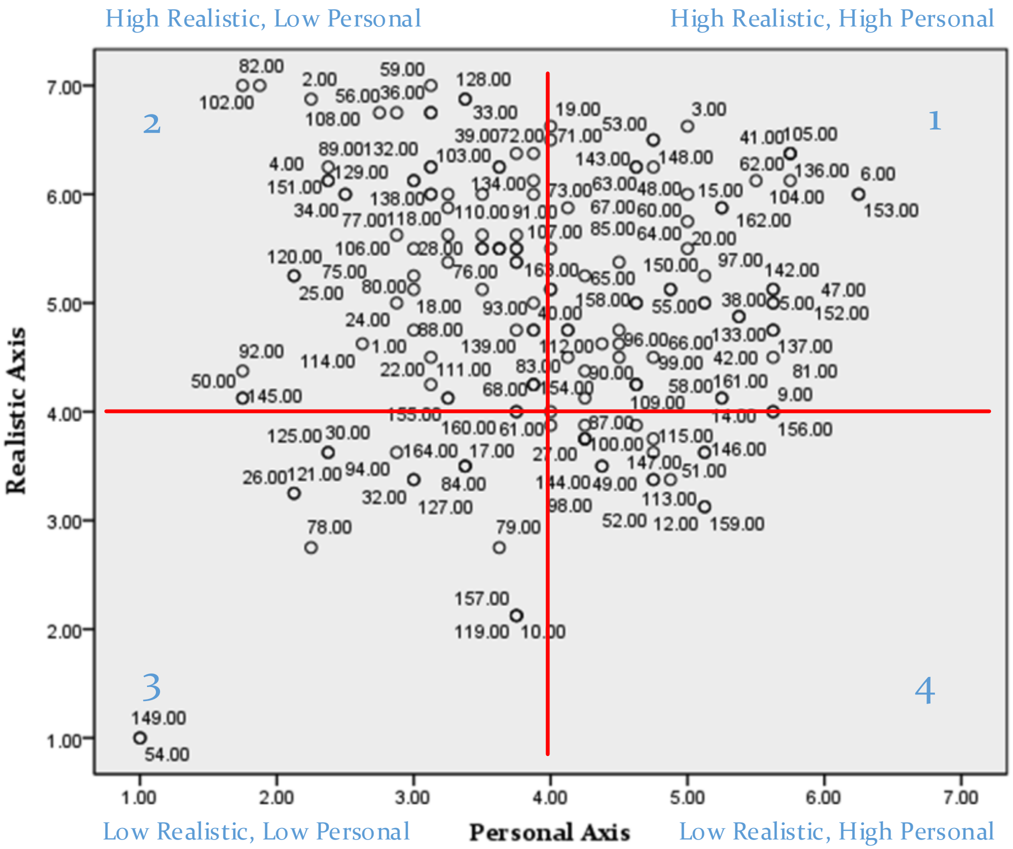


*Figure 9*: The Four Quadrants of Achievement Bests

Source: Phan, H. P., & Ngu, B. H. (In Review). Understanding the Framework of Achievement Bests. *The Journal of Educational Research*.

# References


Phan, H. P., Ngu, B. H., & Williams, A. (2015). *The Optimal Outcomes Questionnaire*. Education. The University of New England. Armidale, Australia.

Phan, H. P., Ngu, B. H., & Williams, A. (2016). Introducing the concept of Optimal Best: Theoretical and methodological contributions. *Education, 136*(3), 312-322.

Phan, H. P., Ngu, B. H., & Yeung, A. S. (In press-2016). Achieving optimal best: The use of cognitive load theory in mathematical problem solving. *Educational Psychology Review*. DOI: 10.1007/s10648-016-9373-3

Vygotsky, L. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.

Wigfield, A., & Eccles, J. S. (2000). Expectancy-value theory of achievement motivation. *Contemporary Educational Psychology, 25*(1), 68-81.